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Technical Note

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Exact Solutions for Instability Control of Piezolaminated Imperfect Struts

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Introduction

FLEXIBLE smart structures, when subjected to compressive loads, are susceptible to form failure. Form failure may be triggered due to imperfections caused by either geometry of the structure or loads on the structure. Euler determined the critical load on a perfect strut by applying a perturbation to it. The method does not predict the lateral deformation of the strut. In practice, however, the struts are not perfect and they must have a finite deformation for varying axial loads as described in the imperfection approach.[1] Most investigations, however, attempt to find the critical buckling load for piezolaminated structures. Several authors reported on theoretical and experimental research on active buckling control of piezolaminated structures following the Eulerian approach.[2,3] This Note presents a comprehensive formulation for instability control of struts using an imperfection approach under arbitrary dynamic excitation with a combination of both displacement feedback and velocity feedback. It is shown that displacement feedback is required to reduce the amplitude of vibration and velocity feedback increases damping in the system. The energy requirement for the different active control methodologies is investigated. Control of steel struts with surface-bonded lead zirconate titanate (PZT) layers under axial arbitrary loading is demonstrated.

Proposed Formulations

In this section we discuss the basic concept for controlling the deflections due to application of mechanical loads using the sensing and actuation mechanisms. When an imperfect strut is subjected to axial compressive loads, it deflects laterally. A large amount of lateral deflection is obtained when the axial load approaches Euler's critical buckling load. To eliminate this deflection and restore the initial imperfection, we need to measure the change in deformation in the strut and apply a remedial voltage that resists the deformation. Figure 1 shows a simply supported strut with geometric imperfection. The initial imperfection of the strut is w_0, and it deflects to w due to the load. For simply supported struts

$$w_0 = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{L}, \qquad w(x,t) = \sum_{n=1}^{\infty} f_n \sin\frac{n\pi x}{L} \tag{1}$$

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where f_n is a function of time t. The sensed voltages at the two layers are equal with opposite polarity. The equation of motion for the strut member can be written as

$$D\frac{\partial^4 w}{\partial x^4} + P(t)\frac{\partial^2}{\partial x^2}(w + w_0) + m\frac{\partial^2 w}{\partial t^2} + c\frac{\partial w}{\partial t} + F_c = 0 \tag{2}$$

where D, m, and c are the flexural rigidity, mass per unit length, and damping coefficient of the strut, respectively; t is the time; and F_c is the control force.

Sensing Strain

Owing to the change in curvature of the strut, there is a change in strain of the piezo layers resulting in generation of electric charge. In the present system the charge is collected only in the thickness direction and the piezoelectric material is poled in the 3-3 direction. The voltage sensed per unit length of the strut can be written as

$$V_S = \frac{q}{C} = \frac{e_{31}\varepsilon b}{\xi_{33}(b/h)} = \frac{e_{31}\varepsilon h}{\xi_{33}} = -\frac{e_{31}[(T+h)/2]h}{\xi_{33}}\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} \tag{3}$$

Here e_{31} is the piezoelectric stress constant, ε is the strain at midthickness of the kth layer, b is the width of the piezo layer, C is the capacitance, and ξ_{33} is the dielectric constant of piezo devices. The sensed current can be written as

$$i_S = \frac{\mathrm{d}q}{\mathrm{d}t} = -e_{31}\left(\frac{T+h}{2}\right)b\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right) \tag{4}$$

In a feedback control system the control forces are proportional to the sensed strains with a gain multiplier. Two different control strategies have been demonstrated here: displacement feedback and velocity feedback.

Actuation

After sensing, the voltage or current actuation is to be applied to control the structure. A simple control strategy is to feed the sensed voltage back with a predetermined gain G_D but with appropriate

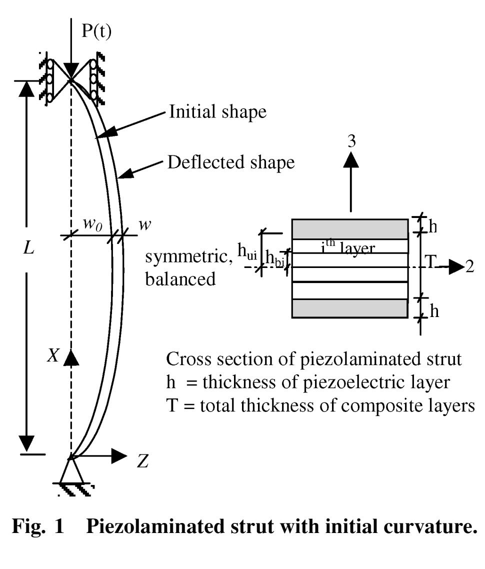

Fig. 1 Piezolaminated strut with initial curvature.

polarity as described in Ref. 4 that is called displacement feedback. It increases the Euler's critical buckling load, thus reducing the amplitude of vibration. Another way of control is to amplify the sensed current with a predetermined gain G_S to enhance the damping to the structure that helps attenuation of the vibration. It is called velocity feedback. Using these control laws the final equation of motion can be written as

$$\bar{D}\frac{\partial^4 w}{\partial x^4} + P_t\frac{\partial^2(w+w_0)}{\partial x^2} + m\frac{\partial^2 w}{\partial t^2} + \left[c\frac{\partial w}{\partial t} + \frac{G_V e_{31}^2 b^2 (T+h)^2}{2Y}\frac{\partial}{\partial t}\left(\frac{\partial^4 w}{\partial x^4}\right)\right] = 0 \tag{5}$$

where $\bar{D}$ is the modified stiffness due to subcritical actuation:

$$D\left[1 + \frac{G_D e_{31}^2 (T+h)^2 hb}{2D\xi_{33}}\right]$$

and $(1/Y)$ is the impedance of the system.

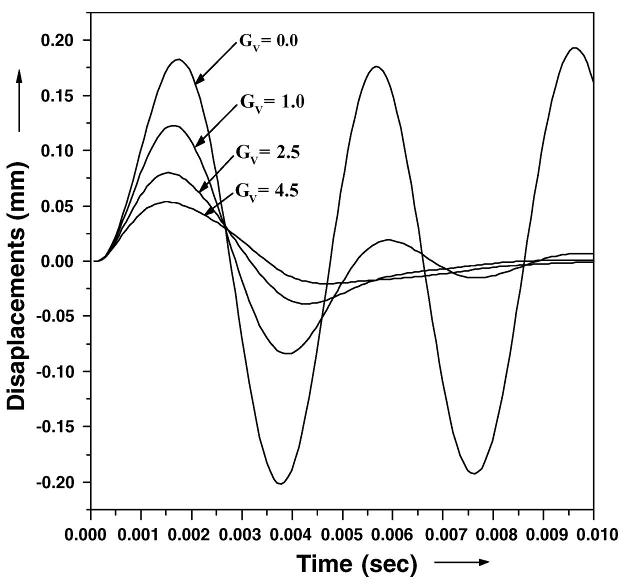

Fig. 2 Active control of strut using velocity feedback only.

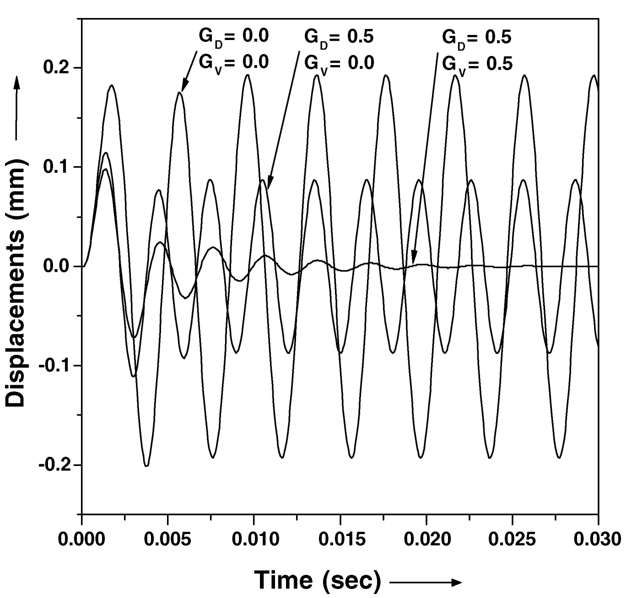

Fig. 3 Constant displacement feedback with varying velocity feedback.

In this control algorithm the power requirements for displacement and velocity feedback are

$$p_D = \frac{1}{2}C\omega G_D^2\frac{e_{31}^2(T+h)^2h^2}{4\xi_{33}^2}\left(\frac{\partial^2 w}{\partial x^2}\right)^2$$

$$p_v = \frac{1}{2}C\omega\frac{G_v^2 e_{31}^2(T+h)^2h^2}{4\xi_{33}^2\omega^2}\left[\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}\right)\right]^2 \tag{6}$$

Numerical Example and Discussion

For the example a simply supported steel strut ($E = 200.0 \times 10^9$ N m^{-2}) surface bonded with PZT is considered ($E = 63.0 \times 10^9$ N m^{-2}, $e_{31} = 6.19$ C m^{-2}, $\xi_{33} = 0.1151 \times 10^{-8}$ F m^{-1}); length of the strut = 100 mm, width of the strut = 5 mm, thickness of the steel layer = 1.0 mm, and thickness of each PZT layer = 0.25 mm. The passive damping of the system is considered as zero to show the damping due to actuation only. The nature of loading is similar to an air blast of duration 0.007 s. The initial imperfection at midheight

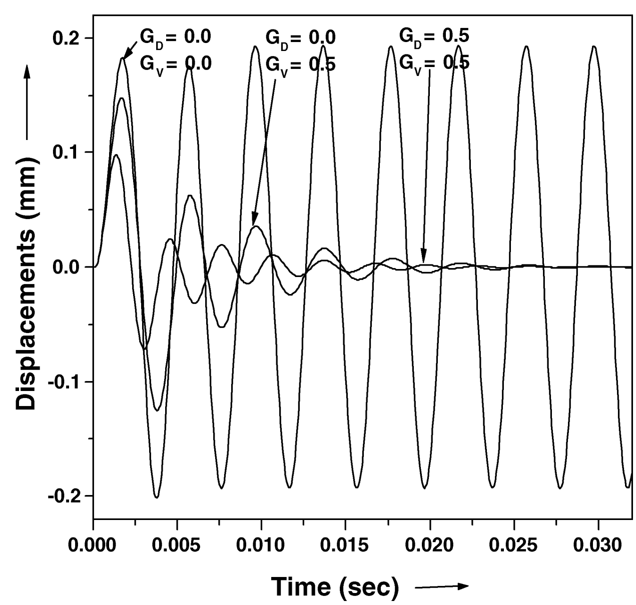

Fig. 4 Constant velocity feedback with varying displacement feedback.

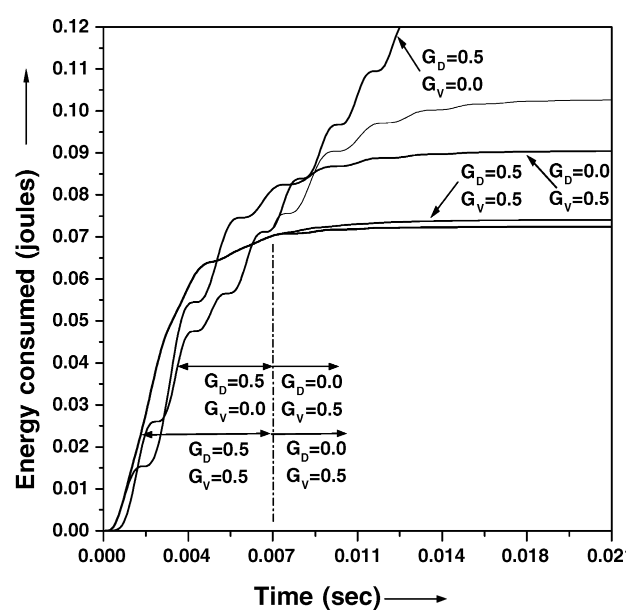

Fig. 5 Consumption of energy for combined displacement and velocity feedback.

is considered as 0.15 mm for $n = 1$. In Fig. 2 it is noted that velocity feedback increases the damping in the structural system. In Figs. 3 and 4 we see the combined effect of displacement and velocity feedback on control of instability or buckling of strut. In the presence of an axial load, displacement feedback reduces the amplitude of vibration. With increasing velocity feedback, the structural vibrations attenuate more quickly due to the active damping imparted into the structure (Fig. 3). With the constant velocity feedback gain the attenuation rate remains constant (Fig. 4), but with increasing displacement feedback gain, the initial amplitude of vibration reduces proportionately. A method of measuring the relative efficacy of the control methodology is the measurement of energy consumption for control of the structure. In Fig. 5 we demonstrate that the energy consumed in combined displacement and velocity feedback is moderate.

Conclusions

The exact solutions for active instability control of strut is demonstrated in this Note by using displacement and velocity feedback methods. A comparative study of the combined displacement and velocity feedback method is shown with respect to energy consumption. Displacement feedback increases the effective stiffness of the structure and velocity feedback augments its damping. Hence, displacement feedback is necessary to reduce the amplitude of vibration and velocity feedback is necessary to attenuate it more rapidly. A judicious selection of displacement and velocity feedback is required for optimum energy consumption in control.

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